CURRENT SYMMETRY CONSIDERATIONS IN THE DESIGN OF THE ATLAS PULSED POWER MACHINE

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Abstract

Two factors that influence the load current symmetry of the Atlas pulsed power machine are analyzed to establish design criteria. Deviations in peak current among the 24 Marx modules providing current to the load must be limited to ±3.1% to achieve the goal of ±100 μm asymmetry in the imploded liner radius. Current asymmetry may also be caused by azimuthal inductance variations in the disk transmission line leading to the load. Mechanical fabrication tolerances are presented that must be met to achieve the symmetry goals.

I. INTRODUCTION

The Atlas pulsed power machine is a very high current, fast discharge capacitor bank presently being constructed [1] at Los Alamos National Laboratory. The principal application of Atlas will be to implode cylindrical metal shells ("liners") to very high velocity so that a variety of high energy density hydrodynamic experiments [2] can be performed. Many of the planned experiments require that the imploding liner accurately maintain its circular cross-section during the implosion. Deviations from circularity can arise from azimuthal variations in the liner density or in the driving current density. Experience with liner implosions on the Pegasus II capacitor bank [3] has clearly demonstrated that material and fabrication tolerances are more than adequate to meet the symmetry requirements. The principal concern therefore is current symmetry.

The current symmetry goal for Atlas derives directly from the implosion symmetry goal. Based on the analysis presented in Section II we expect the dominant liner asymmetry to be a dipole mode, equivalent to a displacement of the liner axis from the target axis. For this asymmetry mode our experimental objectives can be met if the deviation from an ideal circle is less than ±100 μm at a 3:1 convergence (roughly 15 mm radius). To achieve this in a typical 32 MA implosion to 12 km/s the linear current density at the entrance to the load region must be uniform in azimuth to ±0.35%.

Current asymmetry in Atlas will arise from two primary causes, variations in current amplitude among the 24 Marx generators supplying current and deviations from azimuthal symmetry in the disk transmission line carrying current to the load. The effect of current variations in the Marx generators is analyzed in Section II and a design criterion is derived for the Marx generator performance. The problem of transmission line asymmetry is addressed in Section III. Guidelines are developed for the mechanical tolerances that need to be achieved in the fabrication and assembly of the disk transmission line.

II. CURRENT SOURCE VARIATIONS

Current is provided in Atlas by 24 Marx generators, each delivering 1.33 MA with a risetime to peak of 4.5μs. The Marx generators are housed in 12 oil tanks located in a circular pattern approximately 7 meters from the load. Current is carried radially inward from each Marx generator by an oil-insulated, tri-plate transmission line. The 24 tri-plate lines join to a common conductor, called the "transition section," at a radius of 0.8 m. From 0.8 m radius to the load at 0.045 m radius current is carried by a disk transmission line that has both flat and conical regions.

Connections to the transition section are not uniformly spaced. The pair-wise mounting of the Marx generators and the need to provide access between the Marx generator tanks results in the connection geometry shown in Fig. 1. The connection pattern is based on a fundamental increment of 9 degrees, 1/40 of a full circle. The two transmission lines from a common tank are separated by 9 degrees. The tanks are arranged in 4 groups of three. An extra 9 degree space separates the tanks in a group and there is a wider 27 degree separation between groups. Although this connection pattern is not uniform, it has a high degree of symmetry including a

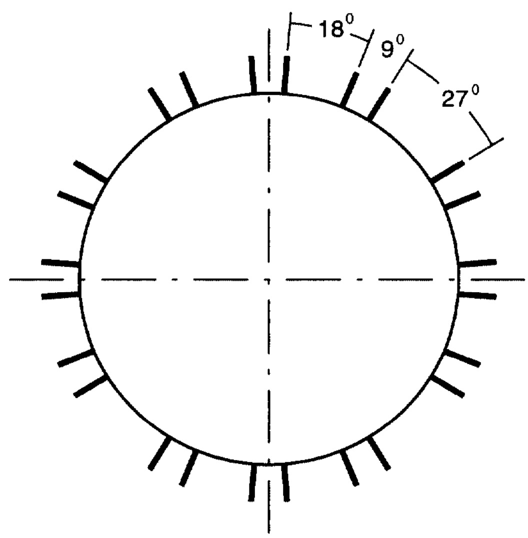

Figure 1. Connection geometry of 24 Marx generators to central transmission line.

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14. ABSTRACT

Two factors that influence the load current symmetry of the Atlas pulsed power machine are analyzed to establish design criteria. Deviations in peak current among the 24 Marx modules providing current to the load must be limited to *3.1% to achieve the goal of +lOO pm asymmetry in the imploded liner radius. Current asymmetry may also be caused by azimuthal inductance variations in the disk transmission line leading to the load. Mechanical fabrication tolerances are presented that must be met to achieve the symmetry goals.

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4-fold rotational axis and two sets of orthogonal mirror planes at 0 and 45 degrees.

A. *Analysis of Radial Current Flow*

In order to relate variations in the Marx generator current amplitude to the symmetry of the load current it is necessary to account for the smoothing effect of the radial current flow in the disk transmission line. For the purposes of this analysis the disk line can be treated as a simple, uniformly spaced pair of flat disks running from 0.80 to 0.045 m radius. This is a reasonably accurate approximation for a conical line if the vertical rise is much less than the radial dimension as it is for Atlas.

Two further approximations can be made that expedite the analysis. First, current flow can be calculated in the quasi-static approximation. The time scale for the Atlas current waveform is about ω^{-1}=3 μs. The transit time of an EM wave through the disk transmission line is only $\Delta r/0.65c$ = 4 ns. The importance of displacement current in determining current flow is roughly proportional to the ratio of these time scales, ~10^{-3}, which certainly justifies the use of the quasi-static approximation.

The second approximation concerns the relative importance of inductive and resistive voltages in determining current flow. For a small radial section of the disk line the ratio of inductive voltage to resistive voltage is

$$\frac{V_{res}}{V_{ind}} = \frac{2\rho}{\mu_o \delta(t) h} \frac{I}{\dot{I}}, \quad (1)$$

where ρ is the conductor resistivity, δ is the skin depth and h is the separation between conductors. During the current rise the quantity I/I-dot ~ t and Eqn. 1 can be further simplified to

$$\frac{V_{res}}{V_{ind}} = \left(\frac{4}{\pi}\right) \frac{2\delta(t)}{h}. \quad (2)$$

At early time the skin depth is small and inductance dominates the current flow. Toward the end of the current rise at t=3 μs the skin depth for an aluminum conductor is 0.3 mm. For a disk spacing of 6.3 mm the voltage ratio is 0.12 so that resistance is still playing a minor role in determining the current distribution. For the remainder of this analysis only inductive effects will be considered.

The quasi-static equations for the magnetic field inside the disk transmission line are $\nabla\times B = \nabla\cdot B = 0$. Using the boundary conditions at the conductor surface, B_z=0, $B_r = \mu_0 J_\theta$, $B_\theta = -\mu_0 J_r$, these equations can be cast in terms of the conductor surface currents.

$$a)\ \frac{\partial r J_\theta}{\partial r} - \frac{\partial J_r}{\partial \theta} = 0 \qquad b)\ \frac{\partial r J_r}{\partial r} + \frac{\partial J_\theta}{\partial \theta} = 0 \quad (3)$$

Eqns. 3 can be solved by separation of variables to yield the following expression for the radial current density.

$$J_r(r,\theta) = \frac{a_0}{r} + \frac{1}{r}\sum_{n=1}^{\infty}\left[(a_n r^n + \frac{c_n}{r^n})\cos(n\theta) + (b_n r^n + \frac{d_n}{r^n})\sin(n\theta)\right] \quad (4)$$

The azimuthal current density has a similar expression that can be derived from Eqn. 4 using Eqn. 3b.

The coefficients {a,b,c,d} are determined by the boundary conditions at the inner and outer radius of the disk, r_i and r_o. At the inner radius the transmission line is short-circuited by the load requiring that $B_r = \mu_0 J_\theta = 0$. The condition $J_\theta(r_i,t) = 0$ requires the following relationship among the coefficients.

$$a_n r_i^n = \frac{c_n}{r_i^n} \qquad b_n r_i^n = \frac{d_n}{r_i^n} \quad (5)$$

The requirement that the integral of rJ_r over all angles is equal to the total current determines $a_0 = I_t/2\pi$. The remaining coefficients must be found by matching the boundary conditions at the outer radius. Denoting the current input from the *jth* source by $I_j(t)$ and the angular position where it is connected by θ_j the radial current density at r_0 is given by

$$J_r(r_0,\theta) = \frac{1}{r_0}\sum_{j=1}^{24} I_j(t)\ \delta(\theta-\theta_j) \quad (6)$$

The coefficients a_n and b_n are found by equating Eqns. 4 and 6, multiplying by sin(mθ) and cos(mθ) and integrating over θ. The result is

$$a_m = \frac{\sum_{j=1}^{24} I_j(t)\cos(m\theta_j)}{\pi\left(r_0^m + \frac{r_i^{2m}}{r_0^m}\right)} \quad (7)$$

and an identical expression for b_m with sin() replacing cos(). Finally, the desired expression for the current density at the load can be obtained by substituting Eqns. 5 and 7 into Eqn. 4.

$$J_r(r_i,\theta) = \frac{I_t}{2\pi r_i}\left\{1 + 4\sum_{n=1}^{\infty}\left(\frac{r_i}{r_o}\right)^n [C_n\cos(n\theta) + S_n\sin(n\theta)]\frac{1}{1+\left(r_i/r_o\right)^{2n}}\right\} \quad (8)$$

where

$$C_n \equiv \frac{1}{I_t}\sum_{j=1}^{24} I_j(t)\cos(n\theta_j) \quad S_n \equiv \frac{1}{I_t}\sum_{j=1}^{24} I_j(t)\sin(n\theta_j).$$

The coefficients (C_n, S_n) are the amplitudes of the n^{th} multipole component of the current excited at the outer radius. Each multipole component appears at the load attenuated by the ratio $(r_i/r_o)^n$. Since $r_i/r_o = 0.056$ for Atlas it is apparent that only the lowest order components will have a significant effect on the load.

B. *Effect of Connection Geometry*

Eqn. 8 can be used to evaluate the effect of the non-uniform connection spacing shown in Fig. 1 for the case where all of the input currents are identical, $I_j = I_t/24$. For the choice of axes in Fig. 1 the θ_j in the first quadrant are [1,5,7,13,15,19] $\pi/40$. The θ_j in the other quadrants are equal to the first quadrant angles plus $\pi/2$, π and $3\pi/2$.

Table 1 displays the values of C_n for n=1 to 40. For the axes chosen all S_n=0. The only non-zero coefficients are those where n is an integer multiple of 4 due to the 4-fold symmetry of the connections.

Table 1. Current distribution coefficients for the case where all input currents are equal.

n	C_n		n	C_n
4	0.121		24	-0.167
8	-0.167		28	-0.513
12	0.513		32	0.167
16	0.167		36	-0.121
20	0		40	-1.000

The largest asymmetry induced at the load by the non-uniform spacing is

$$\frac{\Delta J}{J} = 8\left(\frac{r_i}{r_o}\right)^4 0.121 \cong 9.6\ 10^{-6},$$

much less than the design requirement of 3.5×10^{-3}.

C. *Effect of Current Amplitude Variations*

The 24 current sources that comprise Atlas will not provide identical current amplitudes. One may expect both reproducible differences due to manufacturing tolerances in the components and fluctuating differences due to small variations in charging voltage, spark gap impedance and so forth. In general, the reproducible differences are not of much concern. Once the sources have been calibrated it is relatively simple to arrange them around the disk transmission line so as to minimize excitation of the n=1 mode. For example, the strategy of placing the two highest current sources diametrically opposite each other and then the next two opposite each other and so forth reduces the n=1 coefficient to a negligible level.

Excitation of the n=2 and higher modes is not a concern. Consider, for example, a worst case where 2 of the 24 modules located opposite each other fail to discharge. This does not excite n=1 but it does produce a C_2= 0.08. The resulting load current asymmetry is 2×10^{-3}, well within the design requirement. Since n=2 and higher are so strongly attenuated, only the n=1 mode will be addressed below.

Variations in current amplitude from pulse to pulse can be represented in the form

$$J_r(r_o,\theta) = \frac{I_t}{24 r_o}\sum_{j=1}^{24}(1+\varepsilon_j)\ \delta(\theta-\theta_j), \qquad (9)$$

where the ε_j represent small deviations from the ideal current. In order that I_t be the total current, the ε_j must have a mean value of zero. Prior to operation of the Atlas system there is no information concerning the distribution function for the ε_j. Two alternatives will be considered. In the first case the ε_j are uniformly distributed over an interval $\pm\varepsilon_0$. The second case is a normal probability distribution of width $\pm\varepsilon_n$. Using the current distribution of Eqn. 9 and restricting the analysis to n=1, the load current distribution is given by

$$J_r(r_i,\theta) = \frac{I_t}{2\pi r_i}\left[1 + 4\left(\frac{r_i}{r_o}\right)\left(C_1\cos\theta + S_1\sin\theta\right)\right] \qquad (10)$$

where

$$C_1 = \tfrac{1}{24}\sum_{j=1}^{24} \varepsilon_j\ \cos\theta_j \qquad S_1 = \tfrac{1}{24}\sum_{j=1}^{24} \varepsilon_j\ \sin\theta_j$$

The peak to peak variation of J_r can be found by locating the maximum and minimum of the angular part of Eqn. 10. The final result is

$$\frac{\Delta J_r}{J_r} = 8\left(\frac{r_i}{r_o}\right)\sqrt{C_1^2+S_1^2} \qquad (11)$$

Eqn. 11 can be evaluated statistically to place bounds on the allowable variation in current amplitude. Table 2 presents the results of such a calculation in the form of a probability that the design criterion $\Delta J_r/J_r$ < 3.5x10-3 will be exceeded on a given test as a function of the width of the distribution function. Each probability is calculated by selecting a 24 component ε vector from the appropriate distribution function, calculating C_1 and S_1 according to Eqn. 10 and then comparing the calculated variation, Eqn. 11, to the design criterion. This calculation is carried out N times, where N is 10^5 to 10^6. The total number of times the criterion is exceeded, divided by N, is the probability.

Table 2. Probability of exceeding the load current asymmetry criterion as a function of the variation in current source amplitude.

Uniform distribution			Normal distribution	
ε_0	Prob.		ε_n	Prob.
.025	.0005		.0130	.0005
.028	.003		.0145	.0017
.031	.009		.0160	.005
.034	.021		.0175	.011
.037	.040		.0190	.019
.040	.063		.0205	.033

For an acceptable failure rate of 1 in 100 the amplitude variation must be kept to less than ±3.1% for the uniform distribution. The criterion for the normal distribution is ±1.7%. This appears to be a more stringent requirement. However, the normal distribution has significant population out to 2σ (= 3.4%) so the two criteria are roughly equivalent.

The probability of exceeding the asymmetry criterion is a sensitive function of ε. A 20% increase in ε causes a 4-fold increase in the failure rate. For this reason it will be important during commissioning of Atlas to measure the amplitude variations and identify any modules with excessive variations so that the source can be identified and eliminated.

III. TRANSMISSION LINE ASYMMETRY

The previous section demonstrated the beneficial affect of a converging disk transmission line in smoothing out current asymmetries introduced at large radius. This section will address the complementary effect, the sensitivity of the load current to asymmetries in the disk transmission line.

In the derivation of Eqn. 3 it was assumed that the separation h was independent of θ and r. Allowing h to depend on θ introduces an asymmetry in the disk transmission line inductance that can perturb the symmetry of the load current. The size of this effect can be estimated by noting that for small perturbations, $\Delta h/h \ll 1$, the magnetic field is primarily in the θ direction and that flux conservation requires $B_\theta \cdot h$ = constant. Combining this relation with the boundary condition for B_θ gives for J_r

$$J_r(\theta) \propto {}^{1}\!/_{h(\theta)} \qquad (12)$$

Thus, to satisfy the requirement $\Delta J/J < 3.5\times10^{-3}$ requires $\Delta h/h < 3.5\times10^{-3}$. This is a very stringent mechanical tolerance. For a typical h = 6.35 mm (1/4"), Δh < 0.02 mm (0.001").

A more accurate value for the mechanical tolerance could be obtained by repeating the earlier analysis and including the r and θ dependence of h. However, the solution of this problem is quite involved except for a few special cases and was deemed unnecessary for the present need. Instead, a lumped circuit model of the transmission line was formulated and solved using the circuit code SPICE. The lumped circuit model is shown in Fig. 2. Resistance has been neglect as explained above. Fig. 2 is about the simplest model that can accurately represent a non-symmetric current flow. The angular subdivision into 12 zones allows asymmetries up to n=3 to be investigated. The radial zoning is based on a logarithmic scale so that each zone represents about the same inductance. The choice of 5 zones allows the outer zone to represent the flat portion of the disk line while the inner conical section is resolved into 4 zones.

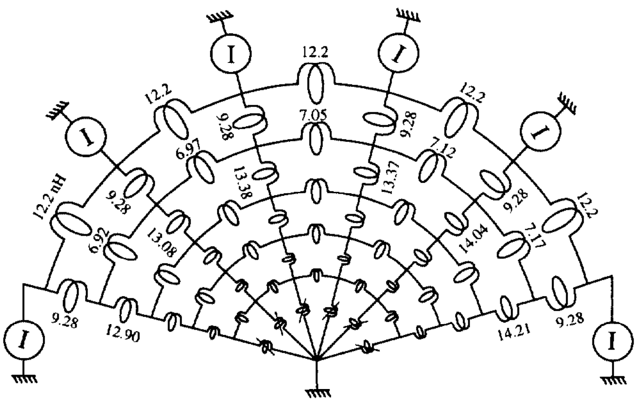

Figure 2. Lumped element model of disk transmission line used to calculated effect of mechanial variations.

The inner edge of the mesh in coupled to inductors that represent the load circuit. Preliminary calculations using a fixed load inductance showed that the current symmetry was sensitive to the size of the load inductance. For greater accuracy a time dependent load inductance was added to the model to approximate the behavior of an imploding liner.

Two of the calculations that were performed with this model are illustrative of the conclusions. In Case 1 the axis of one conductor of the disk line is displaced an amount ε with respect to the other. This produces an n=1 asymmetry in the conical region and has no effect on the flat region. Some typical inductance values for Case 1 with $\varepsilon = 0.07\ h$ are shown in Fig. 2.

The calculation was performed by applying identical current pulses of the form $I_0 \sin(\omega t)$ to each of the outer nodes and measuring the current flowing in each of the load inductors. The asymmetry for Case 1, defined as the maximum difference is current flow at peak current divided by the average current, was 3.4×10^{-2}. Thus, in order to achieve the design goal of $\Delta J/J<3.5\times10^{-3}$ the offset $\varepsilon < 0.007\ h$ or $\Delta h/h < 0.01$. This tolerance is about 3 times larger than the estimate derived above. The primary reason for this increase is the smoothing effect of the load inductance.

For Case 2 one conductor was tilted through a small angle with respect to the other. The asymmetry is n=1 again but the principal effect is at large radius. Summarizing the result, the allowable deviation at the outside radius is $\Delta h/h < 0.04$. The deviation is 4 times greater than for Case 1 because $\Delta h/h$ is small near the load for this geometry and radial convergence can smooth out some of the asymmetry introduced at large radius.

Design of the disk transmission line is proceeding with the objective of achieving $\Delta h/h < 0.005$ in the region from 4.5 to 15 cm radius. Achieving the highest possible symmetry in this region is doubly beneficial. It avoids disturbing the current symmetry and it helps to smooth out asymmetries created at larger radius. Outside of 15 cm radius the allowable $\Delta h/h$ increases linearly up to 0.02 at the 80 cm outer radius of the disk.

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